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Can the Hagedorn phase transition be explained from matrix model for strings?

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Abstract: The partition function of BFSS matrix model is studied for two different classical backgrounds up to 1-loop level. One of the backgrounds correspond to a membrane wrapped around a compact direction and another to a localised cluster of $D0$-branes. It is shown that, there exist phase transitions between these two configurations - but only in presence of an IR cut-off. The low temperature phase corresponds to a string (wrapped membrane) phase and so we call this the Hagedorn phase transition. While the presence of an IR cut-off seemingly is only required for perturbative analysis to be valid, the physical necessity of such a cut-off can be seen in the dual super-gravity side. It has been argued from entropy considerations that a finite size horizon must develop even in an extremal configuration of D0-branes, from higher derivative $O(g_s)$ corrections to super-gravity. It can then be shown that the Hagedorn like transition exists in super-gravity also. Interestingly the perturbative analysis also shows a second phase transition back to a string phase. This is reminiscent of the Gregory-Laflamme instability.

Keywords: Gauge-gravity correspondence, D-branes, M(atrix) Theories.

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Contents

1. Introduction

Hagedorn Temperature [1] is the "limiting temperature" at which the string partition function diverges due to an exponential growth in the density of states, which overtakes the Boltzmann suppression factor. There is some evidence to interpret this temperature as a phase transition temperature [2 – 5]. Yang-Mills theories are known to have confinement-deconfinement phase transition, and also they are known to be dual to String Theories with $1/N$ (for gauge group $SU(N)$) interpreted as the string coupling constant, g_s. It is then

JHEP08(2008)019

natural to identify this Hagedorn Transition temperature with confinement-deconfinement transition [6, 7, 13 – 18]. Above the deconfinement transition the gluon flux tube disintegrates. Correspondingly one would expect that the string disintegrates above the Hagedorn phase transition and is replaced by something else - perhaps a black hole.

It is difficult to study the disintegration of string theory using the perturbative string formalism. One needs a non-perturbative description where a string can be described in terms of some other entity. One such description is the BFSS matrix model. In this model one can construct a classical configuration that looks like a membrane. As a 10-dimensional object it is a D2 brane of IIA string theory. If one of the space dimensions is compactified, the D2 brane wrapped around it, is T-dual to a D-string (D1 brane) of IIB string theory. This in turn is S-dual to an F(fundamental)-string. So on the one hand we can pretend that this D-string is the fundamental string whose phase transition we are interested in, and on the other hand this D-string is (T-dual to) a composite of D0 branes (arranged in a very specific way). At the phase transition this classical membrane configuration can be expected to disintegrate so that we end up with just a bunch of localised D0-branes. This is thus the "S-dual" of the Hagedorn transition. This was what was investigated in in a qualitative way in [8]. It was shown by computing (in a high temperature approximation) the one loop free energy, that there is a phase transition from a membrane phase to a clustered phase. However an IR cutoff was crucial for the calculation. The motivation for the IR cutoff is roughly that the $D0$ branes are actually bound (albeit marginally) and so one expects them to be localised. Two $D0$ brane potential, at finite temperature, studied in [9, 10] shows possibility of bound state. Our aim in this paper is to redo the one-loop analysis carefully without approximations. The net result reaffirms the result of [8], but with some modifications in the analytical expressions. Interestingly an additional phase transition is found *back to the string phase*. The one-loop partition function of strings from matrix model [19] was also studied extensively in [20 – 22].

Holography [44] has given some new insights into the dynamics and phase structure of super-symmetric Yang-Mills [11 – 13]. This should also be taken into account. Deformations of these theories have also been studied [23 – 27]. In fact, motivated by the analysis in [23, 24], the thermal Hawking-Page phase transition [29, 28] in AdS with a "hard wall" [30] has been studied - the hard wall removes a portion of the AdS near $r = 0$, which in the gauge theory corresponds to the IR region. In these models the cutoff is a way to simulate a boundary gauge theory that is not conformally invariant, i.e. confining. The cutoff radius is related to the mass parameter of the gauge theory. It has in fact been shown [31] that an IR cutoff is crucial for the existence of two phases: BPS Dp-branes (more precisely their near horizon limit which is AdS) and black Dp-branes (AdS black hole), separated by a finite temperature phase transition. Also [32] has argued (from entropy considerations) that $O(g_s)$ higher derivative corrections to super-gravity must induce a finite horizon to develop for a configuration of extremal D0-branes. This also acts as an IR cutoff. Based on all this, the main conclusion of this paper is that *an infrared cutoff needs to be included in the BFSS matrix model if it is to describe string theory.*

This is quite understandable from another point of view. Simple parameter counting shows that the BFSS model, as it stands, cannot be equivalent to string theory. It has only

JHEP08(2008)019

one dimensionless parameter N. The other parameter is $g_{\rm YM}$ - which is dimensionful and just sets the overall mass scale. String theory describing N D0 branes has two dimensionless parameter N and g_s, and a dimensionful parameter l_s. The two theories thus cannot be equivalent, except in some limit $g_s \to 0$ (or $g_s \to \infty$ - M-theory). For finite g_s one needs an additional parameter on the Yang-Mills side. Thus an IR cutoff L_0 introduces a second scale into the Yang-Mills theory, and thus the ratio of the two scales is a dimensionless parameter and now the parameter counting agrees. Furthermore, low energies in the matrix model that describes D0 branes, corresponds in the super-gravity description to short distances, i.e. regions near the D0 branes, where the dilaton profile corresponds to a large value of g_s. If we include the low energy (IR) region in the configuration space of the Yang-Mills (matrix model) theory, we are forced to include the effects of finite g_s. The IR cutoff is an additional parameter that signifies our ignorance of (large) strong coupling effects. With an IR cutoff we can maintain g_s at a finite (small) but non-zero value. What actually happens in this region (i.e. close to D0 branes) due to strong coupling effects of large g_s is not fully known, but as mentioned above [32] has a plausible proposal based on entropy arguments. The suggestion is that a finite size horizon develops. This if true, vindicates the introduction of an IR cutoff. Note that another way of making the functional integral finite is to give a mass to the scalar fields - thereby removing the zero mode. The matrix model corresponding to the BMN pp wave limit [33] in fact has such a mass term with the mass being a free parameter. This is reminiscent of the N=1* theories studied in [24] and these techniques have been applied to the BMN matrix model [34]. We can therefore take an agnostic attitude regarding the origin of the cutoff and treat it as the extra parameter necessary to match with string theory for finite g_s. It should correspond to the fact that the D0-brane bound state must have a finite size. This is not easy to see in the BFSS matrix model, in perturbation theory, because of the flat directions in the potential.

As mentioned above there seems to be another high temperature string phase. This is reminiscent of the $T \to 1/T$ symmetry that has been discussed by many authors [35 – 38]. The perturbative analysis shows a second phase transition to a string phase at very high temperatures. This is reminiscent of Gregory-Laflamme instability. We comment on this briefly at the end. We show that the entropic arguments that motivate the Gregory-Laflamme transition can also be made for extremal black holes with finite size horizon such as the Reissner Nordstrom black holes.

This paper is organised as follows. Section 2 is a brief review of some relevant aspects of the BFSS matrix model. Section 3 contains the perturbative one-loop analysis. Section 4 contains the analysis using the super-gravity dual. Section 5 contains some conclusions.

2. M theory

M theory is the strong coupling limit of type IIA string theory. In this limit, it behaves as an eleven dimensional theory in an infinite flat space background. At low energy, it behaves as a eleven dimensional super-gravity. It also has membrane degrees of freedom with membrane tension $\frac{1}{(l_p^{11})^3}$, where l_p^{11} is eleven dimensional Planck length. As mentioned in the introduction these membranes can be wrapped along compact directions to form

JHEP08(2008)019

strings and a study of the partition function of M theory may throw light on Hagedorn Transition, where strings are a particular configuration or phase of more fundamental degrees of freedom.

2.1 BFSS matrix model

M theory in infinite momentum frame (IMF) are described by $D0$-branes only, as proposed in [39]. The action is given by dimensional reduction of 10 dimensional $U(N)$ Super Yang-Mills' (SYM) theory to zero space dimension (in $N \to \infty$ limit). Where the space coordinates of N $D0$-branes are given by eigenvalues of $N \times N$ matrices A_i, $i = 1, \cdots, 9$ of the dimensionally reduced SYM.

Generally the infinite momentum frame is chosen by considering the eleventh direction, X^{11} to be compact (radius, R_{11}) and then subsequently boosting the system in this direction. The negative and zero Kaluza-Klein modes decouple, and can be integrated out. These positive Kaluza-Klein modes have non-zero RR charge from ten dimensional view point (which is type IIA string theory by definition, with $g_s l_s = R_{11}$), and they are identified as the $D0$-branes of the theory. In the end, we must let R_{11} and N/R_{11} tend to infinity to get uncompactified infinite momentum limit.

2.2 DLCQ M-theory

In the method of Discrete Light Cone Quantisation(DLCQ) [41] we compactify a light-like coordinate X^- instead of X^{11}. This theory is valid for any value of N. The idea is that in the large N limit this becomes equivalent to M-theory. For finite N this is a very simple model as shown in [42].

2.3 Relation between DLCQ M-theory and the BFSS matrix model

We will review Seiberg's [42] arguments on the relation between DLCQ M-Theory and BFSS Matrix Model.

In DLCQ, we compactify a light-like circle which corresponds to,

$$\begin{pmatrix} X^{11} \\ X^0 \end{pmatrix} \sim \begin{pmatrix} X^{11} \\ X^0 \end{pmatrix} + \begin{pmatrix} \frac{R^-}{\sqrt{2}} \\ -\frac{R^-}{\sqrt{2}} \end{pmatrix} \tag{2.1}$$

where X^{11} is the longitudinal space-like direction and X^0 is the time-like direction in the 11 dimensional space-time. We can consider it as a limit of compactification on a space-like circle which is almost light like

$$\begin{pmatrix} X^{11} \\ X^0 \end{pmatrix} \sim \begin{pmatrix} X^{11} \\ X^0 \end{pmatrix} + \begin{pmatrix} \sqrt{\frac{R^-}{2} + R_{11}^2} \\ -\frac{R^-}{\sqrt{2}} \end{pmatrix} \simeq \begin{pmatrix} X^{11} \\ X^0 \end{pmatrix} + \begin{pmatrix} \frac{R^-}{\sqrt{2}} + \frac{R_{11}^2}{\sqrt{2}R^-} \\ -\frac{R^-}{\sqrt{2}} \end{pmatrix} \tag{2.2}$$

with $R_{11} \ll R^-$. The light-like compactification (2.1) is obtained from (2.2) as $R_{11} \to 0$. This compactification is related by a large boost with

$$\beta_v = \frac{R^-}{\sqrt{(R^-)^2 + 2R_{11}^2}} \simeq 1 - \left(\frac{R_{11}}{R^-}\right)^2 \tag{2.3}$$

to a spatial compactification on

$$\begin{pmatrix} X^{11'} \\ X^{0'} \end{pmatrix} \sim \begin{pmatrix} X^{11'} \\ X^{0'} \end{pmatrix} + \begin{pmatrix} R_{11} \\ 0 \end{pmatrix} \tag{2.4}$$

where prime denotes boosted coordinates.

The longitudinal boost of the light-like circle (eq. (2.1)) rescales the value of the radius of compactification, $R^{-'} = \delta^{-1} R^-$, where $\delta = \sqrt{\frac{1+\beta_v}{1-\beta_v}}$. It also rescales the value of light-cone energy P^- similarly. Therefore P^- is proportional to R^- i.e. $P^- \sim R^- M_p^2$. The factor of $M_p = (l_p^{11})^{-1}$, the 11 dimensional Planck mass is introduced on dimensional grounds. For small R_{11}, the value of P^- in the system with the almost light-like circle is also proportional to R^- (an exception to that occurs when $P^- = 0$ for the light-like circle; then P^- can be non-zero for the almost light-like circle). The boost (eq. (2.3)) rescales P^- to be independent of R^- and of order R_{11} (if originally $P^- = 0$, the resulting P^- after the boost can be smaller than order R_{11}). So, $P'^- \sim R_{11} M_p^2$, where $R^{-'} = R^{11} = R^-/\delta$.

Let us consider M theory compactified on light-like circle (eq. (2.1)) as the $R_{11} \to 0$ limit of the compactification on an almost light-like circle (eq. (2.2)) or as the limit of boosted circle (eq. (2.4)). Notice $R_{11} \to 0$ corresponds to a large boost, $\beta_v \to 1$ and $\delta \to \infty$. The analysis shows that, the DLCQ of M theory [41] is related to the compactification on a small spatial circle i.e. the BFSS Matrix model [39]. For small R_{11} the theory compactified on (2.4) is weakly coupled string theory with string coupling $g_s = (R_{11} M_p)^{\frac{3}{2}}$ and string length $l_s^2 = (R_{11} M_p^3)^{-1}$. We see in the limit $R_{11} \to 0$ the string length $l_s \to \infty$, which yields a complicated theory. However P'^- also goes to zero (if P^- is initially of $O(1)$), so we are only interested in very low "energy" states of the boosted theory, and this simplifies things. This can be made clear by rescaling parameters. We have to replace DLCQ M theory by another M theory, referred to as $\widetilde{M}$ theory (BFSS Matrix theory) with Planck mass $\widetilde{M_p}$ compactified on the spatial circle of radius R^{11}. The relations between parameters of the two theories is obtained by keeping $P'^- \sim R^{11} \widetilde{M_p}^2$ fixed with the limit $R^{11} \to 0$ and $\widetilde{M_p} \to \infty$, we get,

$$R^{11} \widetilde{M_p}^2 = R^- M_p^2 \tag{2.5}$$

And as boost does not affect transverse directions,

$$M_p R_i = \widetilde{M_p} \widetilde{R_i} \tag{2.6}$$

where R_i are any length parameter in transverse direction. So,

$$\begin{aligned} \frac{\widetilde{M_p}}{M_p} &= \delta^{1/2} \\ \frac{\widetilde{g_s}}{g_s} &= \delta^{-3/4} \\ \frac{\tilde{\alpha}'}{\alpha'} &= \delta^{-1/2} \end{aligned} \tag{2.7}$$

So with finite R^- and M_p, the corresponding string theory in BFSS model is weakly coupled and with very large string tension (Notice as $R_{11} \to 0$ or $\delta \to \infty$, both $\widetilde{g}_s$ and $\widetilde{l}_s$ goes to zero). So this theory is simple.

If we compactify one of the transverse direction with radius R_i and consider the T dual along this direction, the T dual radius is given by $R_i^* = \frac{\alpha'}{R_i}$. So the scaling gives,

$$\widetilde{R}_i^* = R_i^*. \tag{2.8}$$

In this paper we will calculate partition function of the DLCQ theory using the BFSS matrix model. Thus we use parameters (denoted with tilde) which are related to that of DLCQ by a scaling, as discussed above.

2.4 The BFSS matrix model action

The bosonic part of the action for N $D0$-branes is given by,

$$S = \frac{1}{2\widetilde{g}_s} \int \frac{dt}{\widetilde{l}_s} Tr\left\{ (D_t X^i)^2 + \frac{1}{4\pi^2 \widetilde{l}_s^4} [X^i, X^j]^2 \right\} \tag{2.9}$$

where $D_t = \partial_t + iA_0$. Which is basically 10d U(N) SYM action (with $A^i = \frac{X^i}{2\pi\widetilde{l}_s^2}$) reduced to 1d, with 1d $\widetilde{g}_{\rm YM}^2 = \frac{1}{4\pi^2}\frac{\widetilde{g}_s}{\widetilde{l}_s^3}$.

The parameters of DLCQ theory are radius R^- and the eleven dimensional Planck length l_p^{11}. While the parameters of Matrix Model are R^{11} and $\widetilde{l}_p^{11}$. The membrane tension is given by $\frac{1}{(2\pi)^2(\widetilde{l}_p^{11})^3}$. This fixes $\widetilde{g}_s^2 = (\frac{R^{11}}{\widetilde{l}_p^{11}})^3$ and also $\widetilde{\alpha}' = \frac{(\widetilde{l}_p^{11})^3}{R^{11}}$. These relations also imply that $\widetilde{g}_s\widetilde{l}_s = R^{11}$. Where $\widetilde{g}_s$ is the Type IIA string coupling constant and $2\pi\widetilde{\alpha}'$, the inverse string tension with $\widetilde{\alpha}' = \widetilde{l}_s^2$. These parameters are related to that of DLCQ by the scaling discussed.

As mentioned in the introduction, this parameter count in the matrix model is misleading. Written in terms of A_μ it clearly has only one dimensionless parameter N and one scale, set by $g_{\rm YM}$. So the dynamics depends only on N, i.e. all physical quantities will scale with the appropriate power of $g_{\rm YM}$ times some (dimensionless) function of N.

2.5 Construction of membranes and strings

In matrix theory a membrane is described in the large N limit by the configuration [39, 40]

$$X^i = \widetilde{L}^i p\,, \qquad X^j = \widetilde{L}^j q \tag{2.10}$$

where p and q are matrices satisfying,

$$[p, q] = \frac{2\pi i}{N} \tag{2.11}$$

We will consider the configuration given by

$$X^9 = \widetilde{L}^9 p \tag{2.12}$$

which describes a membrane wrapped around X^9 with other edges free. If we consider $X^i, i \neq 9$ as periodic functions of q, all of form $\exp(imq)$, then we get a closed string. Let us construct this string action in matrix model.

Let us assume that X^9 is a compact dimension of radius $\widetilde{L}^9$, assumed to be small. When a membrane is wrapped around X^9 we get a D string(to get F string we have to wrap it around the X^{11}) with inverse tension $\widetilde{\beta}' = \frac{(\widetilde{l}_p^{11})^3}{\widetilde{L}^9}$, and string coupling $\widetilde{g}_{s\beta'} = \frac{(\widetilde{L}^9)^3}{(\widetilde{l}_p^{11})^3}$.

As shown in [8], matrix model action, at zero temperature, with background configuration given by membranes constructed in the above way, matches exactly with the string theory action in light cone frame. The bosonic part of the action used in this paper is given by

$$S = \frac{1}{2\widetilde{g}_s}\int \frac{dt}{\widetilde{l}_s}\int_0^{2\pi\widetilde{L}_9^*}\frac{dx}{2\pi\widetilde{L}_9^*}Tr\left\{(D_tX^i)^2 - (D_xX^i)^2 + (F_{09})^2 + \frac{1}{4\pi^2\widetilde{l}_s^4}[X^i, X^j]^2\right\} \quad (2.13)$$

Which is a $1+1$ dimensional action, obtained by dimensional reduction of $9+1$ dimensional $U(N)$ Super-symmetric Yang-Mills action and subsequently taking T-dual along the compact direction X_9 of radius $\widetilde{L}^9$. $D_x = \partial_x + iA_9$ is the covariant derivative in a direction X_9^* ,which is T-dual to X_9, and has a radius $\widetilde{L}_9^* = \frac{\widetilde{\alpha}'}{\widetilde{L}_9}$. x is the co-ordinate along a $D1$ brane wound around X_9^*.

Following Taylor's [43] calculation we have,

$$A^9 = \frac{1}{2\pi\widetilde{\alpha}'}\sum_{n=-\infty}^{\infty}\exp\left(inx\frac{L_9}{\widetilde{\alpha}'}\right)X_{0n}^9 \quad (2.14)$$

Where $X_{00}^9 = \widetilde{L}^9 p$ is the original $D0$ brane matrix of uncompactified theory. Thus D_x is given by

$$D_x = \partial_x \otimes I + I \otimes \frac{\widetilde{L}^9}{\widetilde{\alpha}' N}\partial_q \quad (2.15)$$

which acts on eigen-functions

$$e^{ir\frac{x}{\widetilde{L}_9^*}}e^{imp}e^{inq} \quad (2.16)$$

with eigen values $\frac{rN+n}{N\widetilde{L}_9^*}$. This action(2.13), in the large N limit, matches with closed string action [8], where the effective radius of world-sheet is $N\widetilde{L}_9^*$ and inverse string tension is $2\pi\widetilde{\beta}'$, $\widetilde{\beta}' = \widetilde{L}_9^* R^{11}$. Turning on F_{09} corresponds to addition of F-strings. The commutator terms are zero if we restrict the matrices X^i to be $X^i(x,q,t)$ i.e. without any p dependence. p dependence corresponds to fluctuations in matrix model that are not string-like.

2.5.1 Two phases

Phase 1: The background $X^9 = \widetilde{L}^9 p$ gives a configuration where the $D0$ branes spread out to form a string wound in the compact direction.
*Phase 2:*The background $X^9 = 0$ gives a phase where the $D0$-branes are clustered.

We will consider these two backgrounds to calculate free energy up to one loop level, and compare to find any signature of phase transition. It is important to have a precise definition of the measure in the functional integral. This is described in appendix A.

3. One loop free energy

For convenience we will drop the tilde sign on the parameters and put it back in the end.

The details are given in the appendices B and C. We summarise the results here. We will first calculate action for a SUSY scalar field on $S^1 \times S^1$, which is then related to the action (2.13) we are concerned in the following subsection. We start with the bosonic part:

Consider the Euclidean action,

$$S = \frac{1}{g_s}\int_0^\beta \frac{dt}{l_s}\int_0^{2\pi L_9^*}\frac{dx}{2\pi L_9^*}\{(\partial_t X)^2 + (\partial_x X)^2\} \tag{3.1}$$

Where t and x directions are both periodic with periods β and $2\pi L_9^*$ respectively, then,

$$X = \sqrt{2\pi L_9^*\beta}\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} X_{nm}e^{-\frac{2\pi i}{\beta}nt}e^{-\frac{i}{L_9^*}mx} \tag{3.2}$$

X is real implies $X^*_{nm} = X_{-n-m}$. So we get,

$$S = \frac{\beta}{2g_s l_s}\sqrt{2\pi L_9^*\beta}\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\left[\left(\frac{2\pi n}{\beta}\right)^2 + \left(\frac{m}{L_9^*}\right)^2\right]X_{nm}X_{-n-m} \tag{3.3}$$

Now we can calculate partition function easily (see appendix B). We get,

$$Z = \frac{L_0}{\sqrt{2\pi g_s l_s \beta}}\eta(i\frac{\beta}{2\pi L_9^*})^{-2} \tag{3.4}$$

where $\eta(x)$ is Dedekind's Eta function and L_0 is cut-off introduced in zero mode integral.

The Partition function in terms of ratio of radii of two S^1, ie. $x = \frac{\beta}{2\pi L_9^*}$,

$$Z = \frac{L_0}{\sqrt{4\pi^2 g_s l_s L_9^*}}\frac{1}{\sqrt{x}}\eta(ix)^{-2} \tag{3.5}$$

Dedekind's Eta Function has a symmetry given by,

$$\eta(ix) = \frac{1}{\sqrt{x}}\eta(i/x) \tag{3.6}$$

Which makes the partition function invariant under the transformation $x \to 1/x$

For low temperature, $\frac{\beta}{2\pi L_9^*} \gg 1$, the free energy takes the form,

$$F(T) = -\frac{1}{\beta}ln(Z) \simeq -\frac{1}{12L_9^*} - \frac{1}{2}Tln\left(\frac{L_0^2}{2\pi g_s l_s}T\right) \tag{3.7}$$

which shows $F(0) \neq 0$ due to the presence of zero-point energy,

$$F(0) = -\frac{1}{12L_9^*} = \sum_{n=1}^{\infty}\frac{n}{L_9^*} \tag{3.8}$$

using Zeta function regularization. The high temperature expansion, $\frac{\beta}{2\pi L_9^*} \ll 1$ is given as,

$$F(T) = -\frac{\pi^2 L_9^* T^2}{3} + \frac{T}{2}ln\left(\frac{8\pi^3 g_s l_s L_9^{*2}}{L_0^2}T\right) \tag{3.9}$$

Now we add in the fermions:

The Minkowaski action is given by,

$$S_M = \frac{-i}{g_s}\int \frac{dt_M}{l_s}\int_0^{2\pi L_9^*}\frac{dx}{2\pi L_9^*}\{(\partial_{t_M}X)^2-(\partial_x X)^2+\bar{\psi}(i\gamma^\mu)\partial_\mu\psi\} \tag{3.10}$$

ψ_α, $\alpha = 1,2$ are two components (real) of two dimensional Maiorana Spinor ψ.

γ matrices are given by,

$$\gamma^0 = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} \tag{3.11}$$

$$\gamma^\mu, \gamma^\nu = 2g^{\mu\nu}, g^{00} = -g^{11} = 1 \tag{3.12}$$

Fermionic part of the action can be rewritten as,

$$\bar{\psi}(i\gamma^\mu)\partial_\mu\psi = i\psi_1(\partial_{t_M}-\partial_x)\psi_1 + i\psi_2(\partial_{t_M}+\partial_x)\psi_2 \tag{3.13}$$

Now to go to Euclidean action at finite temperature we have to take $t_M \to it$,and t compact with periodicity β.We get,

$$S = -\frac{1}{g_s}\int_0^\beta \frac{dt}{l_s}\int_0^{2\pi L_9^*}\frac{dx}{2\pi L_9^*}\{(\partial_t X)^2+(\partial_x X)^2-\psi_1(\partial_t - i\partial_x)\psi_1 - \psi_2(\partial_t+i\partial_x)\psi_2\} \tag{3.14}$$

Where X is periodic in both t and x, ψ_α is anti-periodic in t and periodic in x.

$$X = (2\pi L_9^*\beta)^{1/2}\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} X_{nm}e^{-\frac{2\pi i}{\beta}nt}e^{-\frac{i}{L_9^*}mx} \tag{3.15}$$

$$\psi_\alpha = (2\pi L_9^*\beta)^{1/4}\sum_{n=-\infty,n=odd}^{\infty}\sum_{m=-\infty}^{\infty}\psi_{\alpha,nm}e^{-\frac{\pi i}{\beta}nt}e^{-\frac{i}{L_9^*}mx} \tag{3.16}$$

X and ψ_α is real implies $X^*_{nm} = X_{-n-m}$ and $\psi^*_{\alpha,nm} = \psi_{\alpha,-n-m}$. X_{nm} is a dimensionless c-number and $\psi_{\alpha,nm}$ is a dimensionless Grassmann number.

So the action becomes,

$$\begin{aligned} S = &-\frac{\beta}{2g_s l_s}\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}(2\pi L_9^*\beta)\left[\left(\frac{2\pi n}{\beta}\right)^2+\left(\frac{m}{L_9^*}\right)^2\right]X_{nm}X_{-n-m} \\ &-\frac{\beta}{2g_s l_s}\sum_{n=-\infty,n=odd}^{\infty}\sum_{m=-\infty}^{\infty}\left\{i\sqrt{(2\pi L_9^*\beta)}\left[\frac{\pi n}{\beta}+i\frac{m}{L_9^*}\right]\psi_{1,nm}\psi_{1,-n-m}+\right. \\ &\left. i\sqrt{(2\pi L_9^*\beta)}\left[\frac{\pi n}{\beta}-i\frac{m}{L_9^*}\right]\psi_{2,nm}\psi_{2,-n-m}\right\} \end{aligned} \tag{3.17}$$

For Bosonic part of the action we will get same partition function (Z_B) as previous section. Each ψ_α contributes same amount to partition function, $Z_F = Z_{F1}Z_{F2} = Z_{F1}^2 = Z_{F2}^2$. We get (see appendix C),

$$Z_F = Z_{F\alpha}^2 = 2\frac{\eta(2ix)^2}{\eta(ix)} \tag{3.18}$$

Where $x = \frac{\beta}{2\pi L_9^*}$.

So SUSY partition function,

$$Z = Z_B Z_F = Z = \frac{2L_0}{\sqrt{4\pi^2 g_s l_s L_9^*}} \frac{1}{\sqrt{x}} \frac{\eta^2(2ix)}{\eta^4(ix)} \tag{3.19}$$

Compare with,

$$Z_B = \frac{L_0}{\sqrt{4\pi^2 g_s l_s L_9^*}} \frac{1}{\sqrt{x}} \eta(ix)^{-2} \tag{3.20}$$

Now Z does not have the $x \to 1/x$ symmetry, which is natural as two directions are not similar due to different boundary condition. At low temperature, i.e. $x \to \infty$, $Z = 2L_0\sqrt{\frac{(\frac{1}{g_s l_s})}{2\pi\beta}}$, which is the partition function for a super-symmetric free particle (the zero mode).

3.1 Free energy for two phases of matrix model

We use Background Gauge Fixing Method (see appendix D) to calculate the free energy up to one loop for the action (2.13). The ghost terms effectively cancels the two gauge fields, and remaining theory is effectively that of 8 SUSY scalar fields, except the fields are now $U(N)$ matrices in adjoint representation and the derivatives are little complicated than that for scalar fields. Now in phase 2 (clustered) the covariant derivatives reduces to ordinary derivatives. The partition function for phase 2 (clustered) is,

$$Z_2 = e^{-\beta F_2} = \left\{ \frac{2\widetilde{L}_0}{\sqrt{4\pi^2 \widetilde{g}_s \widetilde{l}_s \widetilde{L}_9^*}} \frac{1}{\sqrt{x}} \frac{\eta^2(2ix)}{\eta^4(ix)} \right\}^{8N^2} \tag{3.21}$$

$$\beta F_2 = -8N^2 ln(b) + 8N^2 ln(\sqrt{x}) - 16N^2 ln(\eta(2ix)) + 32N^2 ln(\eta(ix)) \tag{3.22}$$

The N^2 comes as the eigenvalues are independent of m and n (see eq. (2.16)). For phase 1 (string) the background of A_9 effectively changes the radius $\widetilde{L}_9^*$ to $N\widetilde{L}_9^*$ (see section (2.5)), and the partition function for phase 1 is given by,

$$Z_1 = e^{-\beta F_1} = \left\{ \frac{2\widetilde{L}_0}{\sqrt{4\pi^2 \widetilde{g}_s \widetilde{l}_s N \widetilde{L}_9^*}} \frac{1}{\sqrt{\frac{x}{N}}} \frac{\eta^2(i\frac{2x}{N})}{\eta^4(i\frac{x}{N})} \right\}^{8N} \tag{3.23}$$

$$\beta F_1 = -8N ln(b) + 8N ln(\sqrt{x}) - 16N ln\left(\eta\left(i\frac{2x}{N}\right)\right) + 32N ln\left(\eta\left(i\frac{x}{N}\right)\right) \tag{3.24}$$

The N comes as eigenvalues are independent of m (see eq. (2.16)). Where $x = \frac{\beta}{2\pi\widetilde{L}_9^*}$, $b = \frac{2\widetilde{L}_0}{\sqrt{4\pi^2 \widetilde{g}_s \widetilde{l}_s \widetilde{L}_9^*}} = \frac{2\widetilde{L}_0}{\sqrt{4\pi^2 \widetilde{\beta}'}}$.

3.1.1 Low temperature expansion

Consider $x \gg 1$ and $x/N \gg 1$,

$$\beta F_1 \simeq -8N ln(b) + 8N ln(\sqrt{x}) \tag{3.25}$$

$$\beta F_2 \simeq -8N^2 ln(b) + 8N^2 ln(\sqrt{x}) \tag{3.26}$$

Using,

$$\eta(ix) \simeq e^{-\frac{\pi x}{12}} \text{ for } x \gg 1 \tag{3.27}$$

So $\beta F_1 < \beta F_2$, i.e. the string phase will be favoured at low temperature if $b/\sqrt{x} \leq 1$. We can expect a phase transition from the string phase to the clustered phase as the temperature is increased from zero, at $x = b^2$. For the transition temperature to lie in the validity region of the low temperature expansion: $b \gg \sqrt{N}$. Let us call this temperature as T_H.

$$T_H = \frac{\pi R^{11}}{2\widetilde{L}_0^2} \tag{3.28}$$

In terms of the DLCQ parameters,

$$T_H = \frac{\pi R^-}{2L_0^2} \tag{3.29}$$

using the scaling properties discussed in section (2.3). If we now take the limit $L_0 \to \infty$, the transition temperature $T_H \to 0$. *So, it is essential to have a finite value of L_0 to get phase transition at finite temperature.* This is the temperature at which there is a deconfinement transition in the Yang-Mills' model, which should be same as *Hagedorn transition.*

3.1.2 High temperature expansion

Consider $x \ll 1$,

$$\beta F_1 \simeq 8Nln\left(\frac{2N}{b}\right) - 8Nln(\sqrt{x}) - \frac{2N^2\pi}{x} \tag{3.30}$$

$$\beta F_2 \simeq 8N^2ln\left(\frac{2}{b}\right) - 8N^2ln(\sqrt{x}) - \frac{2N^2\pi}{x} \tag{3.31}$$

Using,

$$\eta(ix) \simeq e^{-(\frac{\pi}{12x} + ln\sqrt{x})} \text{ for } x \ll 1 \tag{3.32}$$

We see that, at $x > \frac{4}{b^2}$, the clustered phase is favoured but at very high temperature we can again have a string phase. This "Gregory-Laflamme" kind of transition will occur at $x = \frac{4}{b^2}N^{-1/N} \simeq \frac{4}{b^2}$ (For large N, $N^{-1/N} \sim 1$). This is also consistent with $b \gg \sqrt{N}$. Let us call this temperature as T_G.

$$T_G = \frac{\widetilde{L}_0^2}{8\pi^3 R^{11}\widetilde{L}_9^{*2}} \tag{3.33}$$

In terms of the DLCQ parameters,

$$T_G = \frac{L_0^2}{8\pi^3 R^- L_9^{*2}} \tag{3.34}$$

using the scaling properties discussed in section 2.3. In this case, note $L_0 \to \infty$ implies $T_G \to \infty$.

Let us express this in terms of the parameters of Yang-Mills theory: The infrared cutoff on $A = \frac{X}{\widetilde{l}_s^2}$ is $\frac{\widetilde{L}_0}{\widetilde{l}_s^2} = \frac{1}{\widetilde{L}_0^*}$. Thus we get (up to factors of 2π)

$$T_G = \frac{\widetilde{l}_s^4}{R_{11}\widetilde{L}_9^{*2}\widetilde{L}_0^{*2}} = \frac{\widetilde{l}_s^3}{\widetilde{g}_s\widetilde{L}_9^{*2}\widetilde{L}_0^{*2}} = \frac{1}{\widetilde{g}_{YM0+1}^2}\frac{1}{\widetilde{L}_9^{*2}\widetilde{L}_0^{*2}} = \frac{1}{\widetilde{g}_{YM1+1}^2}\frac{1}{\widetilde{L}_9^{*}\widetilde{L}_0^{*2}} \tag{3.35}$$

4. Review of super-gravity results

The BFSS matrix model with tilde parameters is supposed to be a non-perturbative description of M-theory (or IIA String Theory). But the idea of gravity dual [44] of Yang-Mills' theories allows us to relate the matrix model to super-gravity (with tilde parameters). One can use this to infer properties of the matrix model.

The Yang-Mills' coupling associated with $D0$ brane action defined in section (2.4) is given by $g^2_{\rm YM} = \frac{1}{4\pi^2}\frac{g_s}{l_s^3} = \widetilde{g}^2_{\rm YM} = \frac{1}{4\pi^2}\frac{\widetilde{g_s}}{\widetilde{l_s}^3}$ is finite in the limit $\widetilde{g_s} \to 0$ and $\widetilde{l_s} \to 0$, as g_s and l_s are finite parameters. This is the decoupling limit discussed in [44, 42, 45]. In this limit, if we also take N large, the theory is dual to Type II super-gravity solution discussed in [44]. Two classical solutions of Type II super-gravity are: (1) the decoupling limit of black $D0$ branes and (2) the BPS $D0$ branes. In a recent study [31] phase transition of these two solutions were discussed. It was shown that the IR cut-off plays a crucial role in phase transition. As mentioned earlier this is motivated by the work of [30, 23, 24]. We will redo the analysis of this paper [31] here and try to explain the physical origin of IR cutoff used in [31]. The super-gravity solutions have tilde parameters, but for convenience we will drop the tilde signs and put them back at the end.

Ideally, we should construct the super-gravity solution corresponding to the wrapped membrane. We reserve this for the future. Here we are only interested in understanding the nature of the phase transition and the role of the IR cutoff, so we will just use the solution for N coincident D0-branes used also in [31]. In the decoupling limit, (with $U = \frac{r}{l_s^2} =$ fixed, where r is radial co-ordinate defined in the transverse space of the brane. U also sets the energy scale of the dual Yang-Mills theory.) the solution for N coincident black $D0$-branes in Einstein frame is given by,

$$ds^2_{\rm Ein} = \frac{\alpha'}{2\pi g_{\rm YM}}\left\{-\frac{U^{\frac{49}{8}}}{(g^2_{\rm YM}d_0N)^{\frac{7}{8}}}\left(1-\frac{U_H^7}{U^7}\right)dt^2 + \frac{(g^2_{\rm YM}d_0N)^{\frac{1}{8}}}{U^{\frac{7}{8}}}\left[\frac{dU^2}{1-\frac{U_H^7}{U^7}} + U^2 d\Omega_8^2\right]\right\}, \quad (4.1)$$

$$e^{\phi} = 4\pi g^2_{\rm YM}\left(\frac{g^2_{\rm YM}d_0N}{U^7}\right)^{\frac{3}{4}}, \quad (4.2)$$

$$F_{U0} = -\alpha'^{\frac{1}{2}}\frac{7U^6}{4\pi^2 d_0 N g^4_{\rm YM}}. \quad (4.3)$$

where $d_0 = 2^7\pi^{9/2}\Gamma(7/2)$ is a constant. Simply setting $U_H = 0$ gives the solution for N coincident BPS D0-branes.

The Euclidean action can be obtained by setting $t = i\tau$. The Euclidean time τ has a period

$$\beta = \frac{4\pi g_{\rm YM}\sqrt{d_0 N}}{7U_H^{\frac{5}{2}}} \quad (4.4)$$

in order to remove the conical singularity. This is the inverse Hawking temperature of the black $D0$ brane in the decoupling limit.

Now the on-shell Euclidean action for the two solutions can be calculated and gives,

$$I_{\rm black} = \frac{7^3}{16}\frac{V(\Omega_8)\beta}{16\pi G'_{10}}\int_{U_H \text{ or } U_{IR}}^{U_{uv}} U^6 dU \tag{4.5}$$

$$I_{bps} = \frac{7^3}{16}\frac{V(\Omega_8)\beta'}{16\pi G'_{10}}\int_{U_{IR}}^{U_{uv}} U^6 dU \tag{4.6}$$

where $G'_{10} = \alpha'^{-7}G_{10} = 2^7\pi^{10}g_{\rm YM}^4$ is finite in the decoupling limit. U_{uv} is introduced to regularise the action and is taken to ∞ in the end. The temperature of BPS branes β' is arbitrary and can be fixed by demanding the temperature of both the solutions to be same at the UV boundary U_{uv}, which gives $\beta' = \beta\sqrt{1-\frac{U_H^7}{U_{uv}^7}}$. U_{IR} is a IR cut-off which removes the region $U < U_{IR}$ of the geometry. The integration in the action starts from U_{IR} for BPS solution and, U_{IR} or U_H for the black brane solution depending on $U_H < U_{IR}$ or $U_H > U_{IR}$ respectively. If we put $U_{IR} = 0$ i.e. in absence of the IR cut-off, comparison of the actions (eqs. (4.5), (4.6)) shows that there is no phase transition, and the black brane phase is always favoured. Let us consider the case $U_H > U_{IR}$,

$$\Delta I_{\rm bulk} = \lim_{U_{uv}\to\infty}(I_{\rm black} - I_{bps}) = \frac{7^2}{16}\frac{V(\Omega_8)\beta}{16\pi G'_{10}}\Big(-\frac{1}{2}U_H^7 + U_{IR}^7\Big) \tag{4.7}$$

Which shows a change in sign as we increase the temperature i.e. U_H (eq. (4.4)). The system will undergo a phase transition ("Hawking-Page Phase transition") from BPS brane to Black brane solution at $U_H^7 = 2U_{IR}^7$. Actually, we should also consider Gibbons-Hawking surface term for careful analysis (as done in [31]) which corrects the transition temperature by some numerical factor given by,

$$\beta_{\rm crit} = \frac{4\pi\widetilde{g}_{\rm YM}\sqrt{d_0 N}}{7(\frac{49}{20})^{5/14}\ \widetilde{U}_{IR}^{5/2}} \tag{4.8}$$

We see a IR cutoff is essential to realize a phase transition, (as $\widetilde{U}_{IR} \to 0$, $\beta_{\rm crit} \to \infty$) so to get confinement-deconfinement phase transition in dual super Yang-Mills theory we have to introduce a IR cutoff.

As mentioned in the introduction, one possible mechanism for the origin of the cutoff for $D0$ branes can be understood from the analysis of [32]. It was shown that the higher derivative corrections to super-gravity introduce a finite horizon area for extremal $D0$ brane solution which is otherwise zero. The multigraviton states (with total N units of momentum in the 11th direction) and the single graviton state seem to both be microstates of the same black hole when interaction effects higher order in g_s are included. Radius of the horizon developed due to higher derivative corrections is $R \sim \widetilde{l}_s\widetilde{g}_s^{1/3}$. So we can get an estimate of IR cutoff by identifying R with the IR cutoff in our case, $\widetilde{U}_{IR} = \frac{R}{\widetilde{l}_s^2} \sim \frac{\widetilde{g}_s^{1/3}}{\widetilde{l}_s} \sim \widetilde{g}_{\rm YM}^{2/3}$, which is finite in the scaling limit. If we plug in this value of $\widetilde{U}_{IR}$ in eq. (4.8), we get

$$\beta_{\rm crit} = \frac{4\pi\sqrt{d_0 N}}{7(\frac{49}{20})^{5/14}\ \widetilde{g}_{\rm YM}^{2/3}} \sim \frac{1}{\widetilde{U}_{IR}} \tag{4.9}$$

JHEP08(2008)019

In case of $D1$ brane system, which is just T dual to the system studied above also shows that a IR cutoff is required for phase transition [31]. We were unable to find a analysis like [32] corresponding to wrapped membrane system, which we need to get an estimate of the IR cutoff.

4.1 Gregory-Laflamme transition

In our calculation we find a temperature T_G, where the $D0$-branes spread out uniformly along the compact space. This configuration is just the one that is favoured at very low temperatures. It is not clear whether this perturbative result is reliable. However, a similar phase transition exists in the dual super-gravity theory, known as "black hole-black string" transition or Gregory-Laflamme transition [11, 12, 46, 47, 49, 50]. It is shown in [11], that the near horizon geometry of a charged black string in $R^{8,1} \times S^1$ (winding around the S^1) develops a Gregory-Laflamme instability at a temperature $T_{GL} \sim \frac{1}{L^2 g_{\rm YM}\sqrt{N}}$, where L is the radius of S^1 and $g_{\rm YM}$ is $1+1$ dimensional Yang-Mills' coupling. Below this temperature the system collapses to a black-hole. In the weak coupling limit, the dual $1+1$ SYM theory also shows a corresponding phase transition by clustering of eigenvalues of the gauge field in the space-like compact direction below the temperature, $T'_{GL} \sim \frac{1}{L^3 g_{\rm YM}^2 N}$, as shown numerically in [11]. This should be compared to the perturbative result (eq. (3.35)) $T_G \sim \frac{1}{g_{\rm YM}^2 (L_0^*)^2 L_9^*}$. So the presence of high temperature string phase in our model must correspond to some kind of "Gregory-Laflamme" transition in dual super-gravity. This is (at least superficially) independent of the issue of any classical instability. This is because both solutions may be locally stable, but at finite temperatures it is possible to have a first order phase transition to the global minimum.

In our perturbative result, the high temperature phase is a string rather than a black string i.e. it is the same as the low temperature phase. The question thus arises whether Gregory-Laflamme transitions can happen for extremal objects. We can give a heuristic entropy argument to show Gregory-Laflamme kind of transition is also possible for extremal system. In the original argument [48], it was shown that for extremal branes there is no instability. However, these systems had zero horizon area. Instead, we will here consider a 5 dimensional extremal RN black hole with a large compact direction, and the same solution with the mass smeared uniformly along the compact direction ("RN black ring"). The metric, ADM mass (M_5) and entropy (S_5) for a $5d$ extremal RN black hole solution is given by (where the compact direction is approximated by a non-compact one),

$$ds_5^2 = -\left(1 - \frac{r_e^2}{r^2}\right)^2 dt^2 + \left(1 - \frac{r_e^2}{r^2}\right)^{-2} dr^2 + r^2 d\Omega_3^2 \tag{4.10}$$

$$M_5 = \frac{3\pi}{4G_5} r_e^2 \tag{4.11}$$

$$S_5 = \frac{2\pi^2 r_e^3}{4G_5} = \frac{\pi^2}{2}\left(\frac{4}{3\pi}\right)^{3/2} G_5^{1/2} M_5^{3/2} \tag{4.12}$$

where G_5 is $5d$ Newton's constant. Similarly, we can write the metric for $5d$ extremal RN Black ring, which when dimensionally reduced gives a $4d$ extremal RN black hole. The

JHEP08(2008)019

metric, ADM mass ($M_{(4\times1)}$) and entropy ($S_{(4\times1)}$) is given by,

$$ds^2_{(4\times1)} = -\left(1-\frac{R_e}{r}\right)^2 dt^2 + \left(1-\frac{R_e}{r}\right)^{-2} dr^2 + dx^2 + r^2 d\Omega_2^2 \tag{4.13}$$

$$M_{(4\times1)} = \frac{R_e}{G_4} \tag{4.14}$$

$$S_{(4\times1)} = \frac{4\pi R_e^2 \times 2\pi L}{4G_5} = \frac{1}{2} G_5 \frac{M^2}{L} \tag{4.15}$$

where $G_4 = \frac{G_5}{2\pi L}$ is $4d$ Newton's constant and L is the radius of the compact direction x. If we consider $M_5 = M_{(4\times1)} = M$ and compare the entropy,

$$\frac{S_5}{S_{(4\times1)}} = \frac{16}{9}\frac{L}{r_e} \tag{4.16}$$

So, when radius of the compact direction is greater than the radius of the $5d$ black hole horizon, the black hole solution is entropically favoured. As we increase the horizon radius, there may be phase transition when horizon size becomes of the order of the radius of the compact dimension, above which the "string" solution is entropically favoured. Our analysis is a simple entropy comparison. As mentioned above, this is independent of the classical stability issue, that was studied in detail in [48]. Therefore, extremal solutions with a *finite horizon size* may also show a Gregory-Laflamme kind of transition. This needs further study.

5. Conclusion

In this paper, the finite temperature phase structure of string theory has been studied using the BFSS matrix model which is a $0+1$ Super-symmetric Yang-Mills (SYM) theory. This was first studied in perturbation theory. This was actually a refinement of an earlier calculation [8] where some approximations were made. The result of this study is that there is a finite and non zero phase transition temperature T_H below which the preferred configuration is where the $D0$ branes are arranged in the form of a wrapped membrane and above which the $D0$ branes form a localised cluster. It is reasonable to identify this temperature with the "Hagedorn" temperature, which was originally defined for the free string. We have found that $T_H \sim \frac{1}{L_0}$, where L_0 is the IR cutoff of the Yang-Mills theory which needs to be introduced to make the calculations well defined.

The $0+1$ SYM has a dual super-gravity description. Here also it is seen that in the presence of an IR cutoff there is a critical temperature above which the BPS D0 brane is replaced by a black hole.

Simple parameter counting shows that the BFSS matrix model needs one more dimensionful parameter if it is to be compared with string theory,so the IR cutoff L_0 can be thought of as one choice for this extra parameter. It makes the comparison well defined by effectively removing the strongly coupled region of the configuration space in SYM as well as in super-gravity. A physical justification for this (beyond parameter counting) comes

JHEP08(2008)019

from the work of [32]. It is shown there that the entropy matching requires even the extremal BPS configuration of D0-branes to develop a horizon, due to higher derivative string loop corrections to super-gravity. This is an issue that deserves further study.

Finally, the perturbative result shows a second phase transition at a higher temperature, back to a string like phase. This could be an artifact of perturbation theory. On the other hand, it is very similar to the Gregory-Laflamme instability and there is also some similarity in the expressions obtained in [11] for the critical temperature. This also requires further study.

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A. Defining measure for $\mathcal{N} = 2$ SUSY in $1D$

We define measure such that

$$\int DxD\psi^* D\psi exp - \pi \int_0^\beta dt \left(\frac{x^2}{2} + \psi^* \psi \right) = 1 \tag{A.1}$$

Where $x(t)$ is a bosonic variable,and $\psi(t)$ is its super-partner.The SUSY transformation is given by,

$$\delta x = \epsilon^* \psi + \psi^* \epsilon; \delta \psi^* = -\epsilon^* x; \delta \psi = -\epsilon x. \tag{A.2}$$

where ϵ^* and ϵ are two infinitesimal anti commuting parameter. From these definition we can define the measure

$$DxD\psi^* D\psi \equiv dx_0 \prod_{n=1}^{\infty} (dx_n dx_{-n}) d\psi_0^* d\psi_0 \prod_{m>0} d\psi_m^* d\psi_{-m}^* d\psi_m d\psi_{-m} \tag{A.3}$$

The Fermionic measure in terms of ψ_1 and ψ_2,where $\psi = \psi_1 + i\psi_2$is given by,

$$D\psi^* D\psi \equiv i d\psi_{10} d\psi_{20} \prod_{m>0} d\psi_{1m} i d\psi_{1m}^* d\psi_{2m} i d\psi_{2m}^* \tag{A.4}$$

where $x(t+\beta) = x(t)$; x_n are Fourier expansion co-efficient for x.ψ_m are Fourier expansion co-efficient for $\psi(t)$,m runs over all integers for periodic boundary condition,but takes only odd values for anti-periodic boundary condition.

$$x(t) = \sum_{n=-\infty}^{\infty} x_n e^{-\frac{2\pi i}{\beta} nt} \tag{A.5}$$

$$\psi(t) = \sum_{n=-\infty}^{\infty} \psi_n e^{-\frac{2\pi i}{\beta} nt} \text{for periodic boundary condition} \tag{A.6}$$

$$= \sum_{n=-\infty,odd}^{\infty} \psi_n e^{-\frac{\pi i}{\beta} nt} \text{for anti-periodic boundary condition} \tag{A.7}$$

A.1 Zeta function

We will need the following results for our calculation,

$$\begin{aligned}
\zeta(s) &= \sum_{n=1}^{\infty} n^{-s} \\
\zeta(s)' &= -\sum_{n=1}^{\infty} n^{-s}\ ln(n) \\
\zeta_{\rm odd}(s) &= \sum_{n=1, n=odd}^{\infty} n^{-s} \\
&= (1-2^{-s})\zeta(s) \\
\zeta_{\rm odd}(s) &= -\sum_{n=1, n=odd}^{\infty} n^{-s}\ ln(n) \\
&= 2^{-s}\ ln2\ \zeta(s) + (1-2^{-s})\zeta(s)'
\end{aligned} \tag{A.8}$$

and $\zeta(0) = -\frac{1}{2}$, $\zeta(0)' = -\frac{1}{2}ln(2\pi)$. Which gives $\zeta_{\rm odd}(0) = 0$ and $\zeta_{\rm odd}(0)' = -\frac{1}{2}ln(2)$.

A.2 Super-symmetric ($\mathcal{N} = 2$) 1D harmonic oscillator

Consider the SUSY Harmonic Oscillator at finite temperature with action,

$$S = \int_0^{\beta} dt \left(\frac{\dot{x}^2}{2} - \psi^* \dot{\psi} + \frac{x^2}{2} + \psi^* \psi \right) \tag{A.9}$$

Where the SUSY transformation is given by,

$$\delta x = \epsilon^* \psi + \psi^* \epsilon; \delta\psi^* = -\epsilon^*(\dot{x} + x); \delta\psi = -\epsilon(-\dot{x} + x). \tag{A.10}$$

With periodic boundary condition on both x and ψ,

$$\begin{aligned}
S = \frac{1}{2}\beta x_0^2 + \beta \sum_{n=1}^{\infty} \left(1 + \frac{4\pi^2 n^2}{\beta^2}\right) x_n x_{-n} + \beta \psi_0^* \psi_0 + \sum_{n=1}^{\infty} (\beta + 2\pi i n) \psi_n^* \psi_n \\
+ \sum_{n=1}^{\infty} (\beta - 2\pi i n) \psi_{-n}^* \psi_{-n}
\end{aligned} \tag{A.11}$$

So integrating by using the measure defined the partition function is,

$$\begin{aligned}
Z &= \int Dx\ D\psi^* D\psi\ e^{-S} \\
&= \sqrt{\frac{2\pi}{\beta}} \times \prod_{n=1}^{\infty} \left\{ \frac{2\pi}{\beta(1 + \frac{4\pi^2 n^2}{\beta^2})} \right\} \times \beta \times \prod_{n=1}^{\infty} (\beta + 2\pi i n) \times \prod_{n=1}^{\infty} (\beta - 2\pi i n) \\
&= 1
\end{aligned} \tag{A.12}$$

Using $\prod_{n=1}^{\infty} C = C^{\zeta(0)} = C^{-1/2}$, where C is any constant number.

JHEP08(2008)019

If we keep periodic boundary condition on x,but take anti-periodic boundary condition on ψ,the Super-symmetry breaks and

$$S = \frac{1}{2}\beta x_0^2 + \beta\sum_{n=1}^{\infty}\left(1+\frac{4\pi^2n^2}{\beta^2}\right)x_n x_{-n} + \sum_{n=1,n=odd}^{\infty}(\beta+\pi in)\psi_n^*\psi_n + \sum_{n=1,n=odd}^{\infty}(\beta-\pi in)\psi_{-n}^*\psi_{-n} \tag{A.13}$$

So integrating by using the measure defined the partition function is,

$$\begin{aligned} Z' &= \int Dx\ D\psi^* D\psi\ e^{-S} \\ &= \sqrt{\frac{2\pi}{\beta}} \times \prod_{n=1}^{\infty}\left\{\frac{2\pi}{\beta(1+\frac{4\pi^2n^2}{\beta^2})}\right\} \times \prod_{m=1,odd}^{\infty}(\beta+\pi im) \times \prod_{m=1,odd}^{\infty}(\beta-\pi im) \end{aligned} \tag{A.14}$$

Now rearranging the products and using ζ function to regularise the infinite products, (i.e. using $\prod_{n=1}^{\infty} C = C^{-1/2}$, $\prod_{n=1}^{\infty} n = \sqrt{2\pi}$, $\prod_{n=1,odd}^{\infty} C = 1$, $\prod_{n=1,odd}^{\infty} n = \sqrt{2}$) we get,

$$Z' = \frac{\prod_{k=0}^{\infty}(1+\frac{4(\beta/2)^2}{\pi^2(2k+1)^2})}{\frac{\beta}{2}\prod_{n=1}^{\infty}(1+\frac{(\beta/2)^2}{\pi^2n^2})} = \coth\beta/2 \tag{A.15}$$

Using,$\prod_{k=0}^{\infty}(1+\frac{4x^2}{\pi^2(2k+1)^2}) = \cosh x$ and $x\prod_{n=1}^{\infty}(1+\frac{x^2}{\pi^2n^2}) = \sinh x$. Also notice as $\beta\to\infty$, $Z'\to 1$ i.e. Super-symmetry is restored in the zero temperature limit.

B. Calculation for massless bosonic field theory on $S^1 \times S^1$

$$S = (2\pi L_9^*\beta)\frac{\beta}{2g_s l_s}\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\left[\frac{4\pi^2n^2}{\beta^2}+\frac{m^2}{L_9^{*2}}\right]X_{nm}X_{nm}^* \tag{B.1}$$

$$\begin{aligned} &= (2\pi L_9^*\beta)\frac{\beta}{g_s l_s}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left\{\left[\frac{4\pi^2n^2}{\beta^2}+\frac{m^2}{L_9^{*2}}\right][X_{nm}X_{nm}^* + X_{n-m}X_{n-m}^*]\right\} \\ &\quad +(2\pi L_9^*\beta)\frac{\beta}{g_s l_s}\sum_{n=1}^{\infty}\frac{4\pi^2n^2}{\beta^2}X_{n0}X_{n0}^* + (2\pi L_9^*\beta)\frac{\beta}{g_s l_s}\sum_{m=1}^{\infty}\frac{m^2}{L_9^{*2}}X_{0m}X_{0m}^* \end{aligned}$$

$$\begin{aligned} Z = &\left(\int_{-\infty}^{\infty}dX_{00}\right)\left(\prod_{n=1}^{\infty}\prod_{m=1}^{\infty}\int_{-\infty}^{\infty}dX_{nm}dX_{nm}^* e^{-(2\pi L_9^*\beta)\frac{\beta}{g_s l_s}\left[\frac{4\pi^2n^2}{\beta^2}+\frac{m^2}{L_9^{*2}}\right]X_{nm}X_{nm}^*}\right) \\ &\left(\prod_{n=1}^{\infty}\prod_{m=1}^{\infty}\int_{-\infty}^{\infty}dX_{n-m}dX_{n-m}^* e^{-(2\pi L_9^*\beta)\frac{\beta}{g_s l_s}[\frac{4\pi^2n^2}{\beta^2}+\frac{m^2}{L_9^{*2}}]X_{n-m}X_{n-m}^*}\right) \\ &\left(\prod_{n=1}^{\infty}\int_{-\infty}^{\infty}dX_{n0}dX_{n0}^* e^{-(2\pi L_9^*\beta)\frac{\beta}{g_s l_s}\frac{4\pi^2n^2}{\beta^2}X_{n0}X_{n0}^*}\right) \\ &\left(\prod_{m=1}^{\infty}\int_{-\infty}^{\infty}dX_{0m}dX_{0m}^* e^{-(2\pi L_9^*\beta)\frac{\beta}{g_s l_s}\frac{m^2}{L_9^{*2}}X_{0m}X_{0m}^*}\right) \end{aligned} \tag{B.2}$$

JHEP08(2008)019

The zero mode integral diverges. So we put a cut-off $\frac{L_0}{\sqrt{2\pi L_9^*\beta}}$ as the value of zero mode integral.

$$Z = \left(\frac{L_0}{\sqrt{2\pi L_9^*\beta}}\right)\prod_{n=1}^{\infty}\prod_{m=1}^{\infty}\left\{\frac{2\pi}{(2\pi L_9^*\beta)\frac{\beta}{g_s l_s}(\frac{4\pi^2 n^2}{\beta^2}+\frac{m^2}{L_9^{*2}})}\right\}^2 \times\prod_{n=1}^{\infty}\left\{\frac{2\pi}{(2\pi L_9^*\beta)\frac{\beta}{g_s l_s}\frac{4\pi^2 n^2}{\beta^2}}\right\}\times\prod_{m=1}^{\infty}\left\{\frac{2\pi}{(2\pi L_9^*\beta)\frac{\beta}{g_s l_s}\frac{m^2}{L_9^{*2}}}\right\} \tag{B.3}$$

Using $\prod_{n=1}^{\infty} c = c^{\zeta(0)} = c^{-1/2}$, if we take out the constant factor $(2\pi L_9^*\beta)$ from the products, it cancels nicely with the factor in zero mode integral. These products can be rearranged to get,

$$Z = \left\{L_0\prod_{n=1}^{\infty}\frac{2\pi}{\frac{\beta}{g_s l_s}(\frac{4\pi^2 n^2}{\beta^2})}\right\}_{\text{free particle with mass}=\frac{1}{g_s l_s}}\times \left\{\prod_{m=1}^{\infty}\left[\sqrt{\frac{2\pi}{\frac{\beta}{g_s l_s}(\frac{m^2}{L_9^{*2}})}}\prod_{n=1}^{\infty}\frac{2\pi}{\frac{\beta}{g_s l_s}\left(\frac{4\pi^2 n^2}{\beta^2}+\frac{m^2}{L_9^{*2}}\right)}\right]_{\text{SHO,mass}=\frac{1}{g_s l_s}\text{freq.}=\frac{m}{L_9^*}}\right\}^2 \tag{B.4}$$

therefore,

$$Z = L_0\sqrt{\left(\frac{M}{2\pi\beta}\right)}\prod_{m=1}^{\infty}\left\{\frac{1}{2\sinh(\beta\omega_m/2)}\right\}^2 \tag{B.5}$$

where,

$$M = \frac{1}{g_s l_s},\ \omega_m = \frac{m}{L_9^*} \tag{B.6}$$

Using,

$$\eta(ix) = \prod_{k=1}^{\infty}(2\sinh(\pi k x)) \tag{B.7}$$

where $\eta(z)$ is Dedekind's eta function. We get,

$$Z = \frac{L_0}{\sqrt{(2\pi g_s l_s\beta)}}\ \eta\left(\frac{i\beta}{2\pi L_9^*}\right)^{-2} \tag{B.8}$$

For low temperature, $\frac{\beta}{2\pi L_9^*} \gg 1$, the free energy takes the form,

$$F(T) = -\frac{1}{\beta}ln(Z) \simeq -\frac{1}{12L_9^*} - \frac{1}{2}T\ ln\left(\frac{L_0^2}{2\pi g_s l_s}T\right) \tag{B.9}$$

which shows $F(0) \neq 0$ due to the presence of zero-point energy,

$$F(0) = -\frac{1}{12L_9^*} = \sum_{n=1}^{\infty}\frac{n}{L_9^*} \tag{B.10}$$

using Zeta function regularization. The high temperature expansion, $\frac{\beta}{2\pi L_9^*} \ll 1$ is given as,

$$F(T) = -\frac{\pi^2 L_9^* T^2}{3} + \frac{T}{2}\, ln\left(\frac{8\pi^3 g_s l_s L_9^{*2}}{L_0^2} T\right) \tag{B.11}$$

C. Calculation for fermionic part of SUSY scalar field theory

$$S_F = S_{F1} + S_{F2} = -\frac{\beta}{2g_s l_s} \sum_{n=-\infty, n=odd}^{\infty} \sum_{m=-\infty}^{\infty} \Big\{ i\sqrt{(2\pi L_9^* \beta)} \Big[\frac{\pi n}{\beta} + i\frac{m}{L_9^*}\Big] \psi_{1,nm} \psi_{1,-n-m} + i\sqrt{(2\pi L_9^* \beta)} \Big[\frac{\pi n}{\beta} - i\frac{m}{L_9^*}\Big] \psi_{2,nm} \psi_{2,-n-m} \Big\} \tag{C.1}$$

By rearranging the sum (we have dropped the index 1 or 2 in ψ),

$$\begin{aligned} S_{F1} = -\frac{\beta}{g_s l_s} \sqrt{2\pi L_9^* \beta} \Big\{ & \sum_{n,m=1,n=odd}^{\infty} \left(\frac{\pi n}{\beta} + i\frac{m}{L_9^*}\right) \psi_{nm}\ i\psi_{-n-m} \\ & + \sum_{n,m=1,n=odd}^{\infty} \left(\frac{\pi n}{\beta} - i\frac{m}{L_9^*}\right) \psi_{n-m}\ i\psi - nm \\ & + \sum_{n=1,odd}^{\infty} \frac{\pi n}{\beta} \psi_{n0} i\psi_{-n0} \Big\} \end{aligned} \tag{C.2}$$

Therefore,

$$Z_{F1} = \Big\{ \prod_{n=1,odd}^{\infty} C\ \frac{\pi n}{\beta} \Big\} \Big\{ \prod_{n=1,odd}^{\infty} \prod_{m=1}^{\infty} C^2 \left(\frac{\pi^2 n^2}{\beta^2} + \frac{m^2}{L_9^{*2}}\right) \Big\} \tag{C.3}$$

where, $C = \frac{\beta}{g_s l_s} \sqrt{2\pi L_9^* \beta}$
Using, $\prod_{n=1,odd}^{\infty} C = C^{\zeta_{\rm odd}(0)} = 1$, $\prod_{n=1,odd}^{\infty} n = e^{-\zeta'_{\rm odd}(0)} = \sqrt{2}$ and rearranging the products we get,

$$Z_{F1} = \sqrt{2} \prod_{n=1,odd}^{\infty} \prod_{m=1}^{\infty} \left(1 + \frac{(\frac{\pi^2 L_9^* n}{\beta})^2}{\pi^2 m^2}\right) \tag{C.4}$$

Using, $\frac{\sinh(x)}{x} = \prod_{k=1}^{\infty}(1 + \frac{x^2}{\pi^2 n^2})$ we get,

$$\begin{aligned} Z_{F1} &= \prod_{n=1,odd}^{\infty} \sinh\left(\frac{\pi^2 L_9^* n}{\beta}\right) \\ &= \frac{\prod_{n=1}^{\infty} 2\sinh(\frac{\pi^2 L_9^* n}{\beta})}{\prod_{n=1}^{\infty} 2\sinh(\frac{2\pi^2 L_9^* n}{\beta})} \\ &= \frac{\eta(i\frac{\pi L_9^*}{\beta})}{\eta(i\frac{2\pi L_9^*}{\beta})} \\ &= \frac{\eta(i\frac{1}{2x})}{\eta(i\frac{1}{x})} \end{aligned} \tag{C.5}$$

Where $x = \frac{\beta}{2\pi L_9^s}$, and using property of Dedekind eta function,we get,

$$Z_{F1} = \sqrt{2}\frac{\eta(2ix)}{\eta(ix)} \tag{C.6}$$

D. Review of background gauge fixing method

Consider dimensionally reduced Maximally Super-symmetric Yang-Mills theory in 10 dimensions to 2 dimension. The Lagrangian is given by,

$$L = \frac{1}{g_{\rm YM}^2} Tr(-(D_\mu A^i)^2 + \theta^T \not{D}\theta - \frac{1}{2}F_{\mu\nu}^2 - \frac{1}{2}[A^i, A^j]^2 + \theta^T \gamma_i [A^i, \theta]) \tag{D.1}$$

Where $i = 1, \ldots 8$ and $\mu, \nu = 0, 9$. The metric is $\eta_{\mu,\nu} = (-1, 1, 1 \ldots, 1)$. θ is 16 component Maiorana Spinor and γ matrices obey 10-dimensional Clifford Algebra. Let us consider,

$$\begin{aligned} A_\mu &= a_\mu + A'_\mu \\ A_i &= a_i + A'_i \\ \theta &= \Theta + \theta' \end{aligned} \tag{D.2}$$

Where a_μ, a_i, Θ are background fields obeying classical equation of motion. Let us define a new covariant derivative as $\bar{D}_\mu = \partial_\mu + i a_\mu$. The primed fields are quantum fluctuations which is integrated out for calculation of partition function. Also,

$$F_{\mu\nu} = \bar{F}_{\mu\nu} + (\bar{D}_\mu A'_\nu - \bar{D}_\nu A'_\mu) + i[A'_\mu, A'_\nu] \tag{D.3}$$

where, $\bar{F}_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu + i[a_\mu, a_\nu]$.

Now the allowed gauge transformation is the ones which keep the background unchanged, i.e. $\delta a_\mu = \delta a_i = \delta\Theta = 0$. Then the gauge transformation on the fluctuations are given by,

$$\begin{aligned} \delta A'_\mu &= \bar{D}_\mu \alpha + i[A'_\mu, \alpha] \\ \delta A'_i &= i[A'_i, \alpha] \\ \delta\theta' &= i[\theta', \alpha] \end{aligned} \tag{D.4}$$

Let us choose $a_i = 0$ and $\Theta = 0$.

The gauge fixing condition we use is,

$$\bar{D}_\mu A^{\mu\prime} = 0 \tag{D.5}$$

therefore the Gauge fixing Lagrangian,

$$L_{gf} = -\frac{1}{2g_{\rm YM}^2} Tr(\bar{D}_\mu A^{\mu\prime})^2 \tag{D.6}$$

and the ghost Lagrangian,

$$L_{gh} = Tr(\bar{\omega}\bar{D}_\mu \bar{D}^\mu \omega + i\bar{\omega}\bar{D}_\mu [A^{\mu\prime}, \omega]) \tag{D.7}$$

JHEP08(2008)019

Where ω and $\bar{\omega}$ are ghost and anti ghost respectively. The background for ghost fields are taken to be zero.

Now we calculate the Lagrangian up to 1-loop level,i.e. we keep terms up to quadratic in fluctuations. And we also use classical equation of motion for background fields. We get,

$$L_{1loop} = \frac{1}{2g_{\rm YM}^2} Tr\Big(A'^i \bar{D}^2 A'^i - A'_0 \bar{D}^2 A'_0 + A'_9 \bar{D}^2 A'_9 + \theta'^T \bar{\not{D}} \theta' - \frac{1}{2}\bar{F}_{\mu\nu}^2 - i\bar{F}^{\mu\nu} A'_\mu A'_\nu \Big) + \bar{\omega}\bar{D}^2\omega \tag{D.8}$$

Let us choose $a_0 = 0$ and $a_9 = constant$, then $\bar{F}^{\mu\nu} = 0$. Also scale ω properly, we get,

$$L_{1loop} = \frac{1}{2g_{\rm YM}^2} Tr(A'^i \bar{D}^2 A'^i - A'_0 \bar{D}^2 A'_0 + A'_9 \bar{D}^2 A'_9 + \theta'^T \bar{\not{D}} \theta' + \bar{\omega}\bar{D}^2\omega) \tag{D.9}$$

The Euclidean partition function is given by,

$$lnZ = \frac{10}{2} Tr(ln\bar{D}^2)_{\rm bosonic} - \frac{16}{4} Tr(ln\bar{D}^2)_{\rm fermionic} - Tr(ln\bar{D}^2)_{\rm ghost} \tag{D.10}$$

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